

(Time: 3 Hours )

Max. Marks: 80

- N.B. (1) Question No. 1 is compulsory.  
 (2) Answer any three questions from Q.2 to Q.6.  
 (3) Use of Statistical Tables permitted.  
 (4) Figures to the right indicate full marks

Q1.

- (a) Find the Laplace transform of  $\frac{\cos 2t \sin t}{e^t}$  [ 5 ]  
 (b) Find k such that  $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{kx}{y}$  is analytic [ 5 ]  
 (c) Calculate the Spearman's rank correlation coefficient R [ 5 ]  
 X : 10, 12, 18, 18, 15, 40.  
 Y : 12, 18, 25, 25, 50, 25.  
 (d) Find the inverse Laplace transform of  $\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$ . [ 5 ]

Q2.

- (a) A continuous random variable has probability density function  

$$f(x) = k(x - x^2), \quad 0 \leq x \leq 1.$$

$$f(x) = 0 \quad \text{otherwise}$$
 Find k, mean and variance. [ 6 ]  
 (b) Find the Laplace transform of  $e^{-3t} \int_0^t u \sin 3u \, du$ . [ 6 ]  
 (c) Obtain the Fourier series to represent  $f(x) = x^2$  in  $(0, 2\pi)$   
 Hence show that  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  [ 8 ]

Q3.

- (a) If the imaginary part of the analytic function  $w = u + iv = f(z)$  is [ 6 ]  
 $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$ , then show that  $u = -2xy + \frac{y}{x^2 + y^2}$ .  
 (b) Find inverse Laplace transform of  $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$  [ 6 ]  
 (c) Fit a second-degree parabolic curve and estimate y when  $x = 10$   

x	: 1, 2, 3, 4, 5, 6, 7, 8, 9,	
y	: 2, 6, 7, 8, 10, 11, 11, 10, 9.	[ 8 ]

Q4.

- (a) Obtain the Fourier series to represent  $f(x) = x^3$  in  $(-\pi, \pi)$ . [ 6 ]  
 (b) Find (i) the equation of the lines of Regression (ii) coefficient of correlation for the following data  

X:	65, 66, 67, 67, 68, 69, 70, 72.	
Y:	67, 68, 65, 66, 72, 72, 69, 71.	[ 6 ]

 (c) Prove that  $\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sin ht}{t} \, dt = \frac{\pi}{8}$ . [ 8 ]

Q5.

(a) Find the orthogonal trajectories of the family of curves  $x^3y - xy^3 = c$ . [6]

(b) Find the moment generating function of the distribution

X	:	-2	3	1
P ( X = x )	:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

hence find first four central moments. [6]

(c) Obtain the half range cosine series of  $f(x) = x$  in  $(0, 2)$

Hence show that  $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$  [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of  $\left[ \frac{s^2}{(s^2+2^2)^2} \right]$  [6]

(b) The probability density function of a random variable X is

X	:	1	2	3	4	5	6	7
P ( X = x )	:	k	2k	3k	$k^2$	$k^2 + k$	$2k^2$	$4k^2$

Find  $k$ ,  $p(X < 5)$ ,  $P(X > 5)$  [6]

(c) If  $v = 3x^2y + 6xy - y^3$ , show that v is harmonic function

And find the corresponding analytic function. [8]

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N.B.

- 1) Q.1 is compulsory.
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

- a) Prove using Mathematical Induction that  $2 + 5 + 8 + \dots + (3n-1) = n(3n+1)/2$  5
- b) Explain the term poset. Consider a set  $D_{165}$ . Find the elements of this set & draw the hasse diagram for this poset. 5
- c) How many strings of length 7 either begin with 2 zeros or end with 3 ones? 5
- d) Explain the term partition set with suitable example. 5
- e) State the Pigeonhole principle and show that If there are 10 marbles in the jar & you have a jar filled with red, green, and blue marbles, you'll always have at least two marbles of the same colour. 5

Q.2

- a) 10
  - Let  $A = \{0, 1, 2, 3, 4, 5\}$
  - i) Explain the term group.
  - ii) Prepare the composition table for the above set w.r.t. the operation of addition modulo 6.
  - iii) Determine whether it is a group.
  - iv) Whether elements of set A are invertible? If yes, then find the inverses of these elements.
  - v) Determine whether it is a cyclic group.

b

Let  $A = \{a_1, a_2, a_3, a_4, a_5\}$  and let R be a relation on A whose matrix is : 10

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find out transitive closure of R using Warshall's algorithm.

Q.3

- a) A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.

Determine the number of computer programmers that are not proficient in any of these three languages. 8

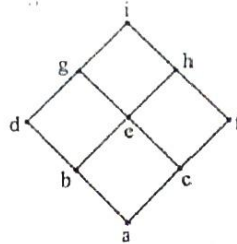
b) Explain the terms Conjunctive & Disjunctive Normal Form with suitable examples. 6

c) 6

Determine the sequence  $b_n$  whose recurrence relation is  $b_n = 2b_{n-1} + 1$  with initial condition  $b_1 = 7$ .

**Q.4**

a) What is a lattice? Determine whether following hasse diagram represents a lattice. 6



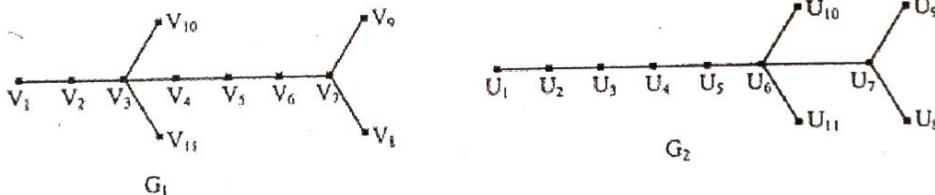
b) 6

Consider  $(3, 8)$  an encoding function  $e : B^3 \rightarrow B^8$  defined as

- $e(000) = 00000000$
- $e(001) = 10111000$
- $e(010) = 00101101$
- $e(011) = 10010101$
- $e(100) = 10100100$
- $e(101) = 10001001$
- $e(110) = 00011100$
- $e(111) = 00110001$

How many errors can 'e' detect & correct?

c) What are the necessary conditions for the isomorphism between 2 graphs? Determine whether following 2 graphs are isomorphic. 8



**Q.5**

a) If the addition & multiplication modulo 10 is defined on a set of integers  $A = \{0, 2, 4, 6, 8\}$ . Then determine whether this algebraic system is a ring. 8

b)

A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2$

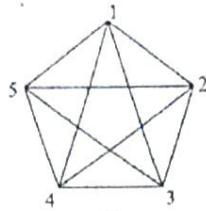
Is it i) injective      ii) surjective      iii) bijective

6

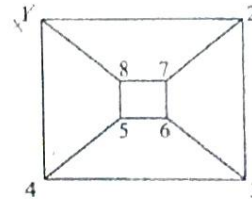
c) Define the terms Euler path & a circuit .

6

Determine whether following graphs have Euler path or a circuit.



a)



b)

Q.6

a) Explain the following terms with suitable example (any 4)

8

- i) Hamiltonian path & circuit
- ii) Bipartite graph
- iii) Adjacency matrix
- iv) Equivalence relation
- v) Cartesian product

b) Solve the following using the laws of logic

4

$$p \vee q \vee (\sim p \wedge \sim q \wedge r) \leftrightarrow p \vee q \vee r$$

c)

8

$f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^3$

$g: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $g(x) = 4x^2 + 1$

$h: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $h(x) = 7x - 2$

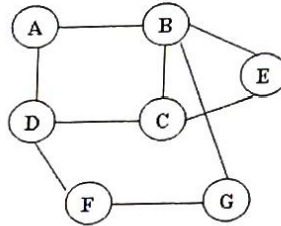
Find the rule defining

- i) fog      ii) gof
- iii) (goh)of      iv) go(hof)

Duration: 3 Hours

- N.B: (1) Question No. 1 is compulsory.  
(2) Attempt any three questions out of the rest.  
(3) Figures to the right indicate full marks.  
(4) Make suitable assumptions wherever necessary with proper justification.

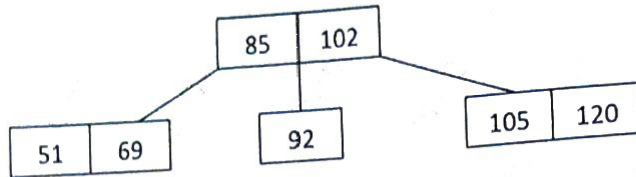
- Q.1. A) Define ADT with an example. [05]  
B) Evaluate the postfix expression "94\*28+-" using stack ADT. Show the process stepwise. [05]  
C) Justify the statement with suitable example: "Circular queue overcomes the disadvantage of linear queue". [05]  
D) Differentiate between linear search and binary search. [05]
- Q.2. A) Construct Huffman tree and determine the code for each symbol in the string "BCAADDCCACACAC". [10]  
B) Discuss the cases of deleting a node from Binary Search Tree with suitable example. [10]
- Q.3. A) Write a program in C to implement queue ADT using linked list. [10]  
B) Construct an AVL tree by inserting the following elements in the given order. Apply necessary rotations wherever required. [10]  
54, 12, 24, 68, 85, 99, 42, 27, 87, 80
- Q.4. A) Write C function for BFS graph traversal. Show the stepwise BFS traversal with the help of data structures for the following graph: [10]



- B) Write functions in C to perform the following operations on the Doubly Linked List: [10]  
i) Delete a node after given node.  
ii) Find node with smallest data value.  
iii) Display the list.  
iv) Insert a node at the end of the list.
- Q.5. A) Build a Binary Search Tree, given the following sequences: [05]  
Inorder: 35, 41, 48, 52, 57, 72, 79, 85, 86, 90  
Preorder: 57, 41, 35, 52, 48, 90, 72, 85, 79, 86  
B) What is topological sort? Explain Topological Sorting with an example. [05]

C) What is collision? Using linear probing, insert the following values in the hash table of size 11 & count the no. of collisions:  
83, 53, 64, 25, 39, 96, 12, 71. [10]

- Q.6. A) Write short note on Priority Queue. [05]  
B) Write a function in C to count the number of nodes in Singly Linked List. [05]  
C) Create a B-tree of order 3 by inserting 87, 94, 59, 98, 63, 7, 27. [10]



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**Time: 3 Hours**

**Max. Marks: 80**

**Instructions:**

- 1) **Question Number 1 is compulsory.**
- 2) Solve any **three** questions out of remaining **five** questions.
- 3) Each Question carry 20 marks.
- 4) Illustrate your answers with neat sketches wherever necessary.
- 5) Figures to the right indicate full marks.
- 6) Assume suitable additional data, if necessary and clearly state it.
- 7) All sub-questions of the same question should be grouped together.

- Q.1** (a) i.) Simplify the Boolean expression:  $A \text{ AND } (B \text{ OR } (C \text{ AND } D))$  using Boolean algebra rules. **03**  
ii.) Create a truth table for the following circuit:  $A \text{ AND } (B \text{ OR } C)$ . **02**
- (b) Convert the IEEE-754 single-precision representation **0 1000010 0101110000000000000000** to its decimal equivalent. **05**
- (c) Discuss the significance of Decoders in address decoding. Provide the truth table for a 3-to-8 Decoder. **05**
- (d) Draw and explain Microinstruction sequencing organization. **05**
- Q.2** (a) A block-set associative cache memory consists of 128 blocks divided into four block sets. The main memory consists of 16,384 blocks and each block contains 256 eight-bit words. **10**  
i.) How many bits are required for addressing the main memory?  
ii.) How many bits are needed to represent the TAG, SET and WORD fields?
- (b) What is bus arbitration? Explain any two techniques of bus arbitration? **10**
- Q.3** (a) Draw and explain the operation of a Master-Slave J-K Flip-Flop with PRESET and CLEAR. How does it differ from a regular J-K flip-flop? **10**
- (b) Explain the concept of a microprogrammed control unit and compare it with a hardwired control unit. Describe the advantages and disadvantages of using a microprogrammed control unit. **10**
- Q.4** (a) Explain how the NAND gate can be used as a universal logic gate. Provide examples of how it can be used to implement other logic gates. **10**
- (b) How Booth's multiplication algorithm can be used to multiply  $(-10)_{10}$  and  $(-7)_{10}$  binary numbers. Show the intermediate steps involved in the multiplication process and explain how the final result is obtained. **10**



- Q.5 (a)** Perform the following binary arithmetic operations and show the intermediate steps and the final result. 10
- i.) Add the following Binary Coded Decimal (BCD) numbers:  
(0101) + (1001).
  - ii.) Subtract the following binary numbers using 2's complement representation: (10101) - (01110).
  - iii.) Multiply the following binary numbers using 1's complement representation: (1101) \* (1010).
  - iv.) Divide the following binary numbers using 2's complement representation: (101101) / (110).
  - v.) Perform addition in hexadecimal for the numbers: (2A) + (1B).
- (b)** What is Pipeline Hazard? Give the types of pipeline hazards. Write a difference between delayed branch and branch prediction. 10
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- Q.6 (a)** Draw instruction cycle state diagram with interrupt. 05
- (b)** What is State Table Method used for design Hardwired Control unit? 05
- (c)** Compare with suitable parameters SRAM with DRAM. 05
- (d)** Draw the neat block diagram for Flynn's classification. 05
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- N.B. 1. Question No. 1 is compulsory  
2. Attempt any 3 from remaining questions.  
3. Assume any suitable data if necessary and justify the assumptions.

Q.1 Attempt any Four 4x5=20

1. Compare DDA and BRESENHAM line drawing algorithm.
2. Give application of computer graphics.
3. Explain with neat diagram rasterization.
4. Give fractal dimension of KOCH curve.
5. Define Projection. Describe perspective projection with neat diagram.

Q.2 20

1. Given a triangle ABC where A(0,0), B(10,10) and C(20,0), scale the given triangle ABC 2-unit in X direction and 0.5-unit in Y direction. Find out the new coordinate of triangle ABC after scaling.
2. Explain with neat diagram Sutherland and Hodgman polygon clipping algorithm in detail.

Q.3 20

1. Derive window to viewport coordinate transformation.
2. Give properties of Bezier curve.

Q.4 20

1. Derive Mid-point circle generation algorithm.
2. Give principles of animation

Q.5 20

1. Explain with neat diagram Area Sub division (Warnock's) algorithm to remove hidden surfaces.
2. Derive matrix for 2D rotation transformation.

Q.6 Attempt any Four 20

1. Explain point clipping algorithm.
2. Give pseudo code for 4-connect Boundary fill algorithm.
3. Give transformation matrix for 3D – Translation, Scaling, Rotation (about x, y, z axis)
4. Explain with neat diagram composite transformation for scaling.
5. Given a line AB where A(0,0) and B(1,3) find out all the coordinate of line AB using DDA algorithm.

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